

Gravitational corrections to Standard Model vacuum decay

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Abstract

We refine and update the metastability constraint on the Standard Model top and Higgs masses, by analytically including gravitational corrections to the vacuum decay rate. Present best-fit ranges of the top and Higgs masses mostly lie in the narrow metastable region. Furthermore, we show that the SM potential can be fine-tuned in order to be made suitable for inflation. However, SM inflation results in a power spectrum of cosmological perturbations not consistent with observations.

1 Introduction

Assuming that the Standard Model (SM) holds up to some high energy scale close to $M_{\text{Pl}} = 1.22 \times 10^{19} \times \text{GeV}$, present data suggest a light Higgs mass, $m_h \sim (115 - 150) \text{ GeV}$. If the Higgs is so light, radiative corrections induced by the top Yukawa coupling can destabilize the Higgs potential and the electroweak vacuum becomes a false vacuum, which sooner or later decays [1, 2, 3, 4, 5, 6]. Demanding that the SM vacuum be sufficiently long lived with respect to the age of the universe implies a bound on the Higgs and top masses [2, 3, 4, 5].

In section 2 we recall the peculiarities of vacuum decay within the SM relevant for our later inclusion of gravity, which was neglected in previous analyses. In section 3 we show how gravitational corrections to the vacuum decay rate [7] can be computed by making a perturbative expansion in the Newton constant, and we obtain the analytic result for SM vacuum decay.

In section 4 we show that for fine-tuned values of the Higgs and top masses (that lie within the experimentally allowed range), the SM potential can be suitable for inflation. However, the corresponding power spectrum of anisotropies is larger than the observed one.

2 Vacuum decay within the Standard Model

We recall vacuum decay within the Standard Model without gravity, and its peculiarities relevant for our later inclusion of gravity. The SM contains one complex scalar doublet H ,

$$H = \begin{bmatrix} (h + i\eta)/\sqrt{2} \\ \chi^- \end{bmatrix}, \quad (1)$$

with tree-level potential

$$V = m^2|H|^2 + \lambda|H|^4 = \frac{1}{2}m^2h^2 + \frac{1}{4}\lambda h^4 + \dots \quad (2)$$

where the dots stand for terms that vanish when the Goldstones η, χ^- are set to zero. With this normalization, $v = (G_F\sqrt{2})^{-1/2} = 246.2 \text{ GeV}$, and the mass of the single physical degree of freedom h is $m_h^2 = V''(h)|_{h=v} = 2\lambda v^2$. As is well known, for $h \gg v$ the quantum corrections to $V(h)$ can be reabsorbed in the running coupling $\lambda(\bar{\mu})$, renormalized at a scale $\bar{\mu} \sim h$. To a good accuracy, $V(h \gg v) \approx \lambda(h)h^4/4$ and the instability occurs if λ becomes negative for some value of h . For the values of m_h compatible with data this occurs at scales larger than 10^5 GeV , suggesting that we can compute vacuum decay neglecting the quadratic term $m^2h^2/2$.

The bounce [1] is a solution $h(r)$ of the Euclidean equations of motion that depends only on the radial coordinate $r^2 \equiv x_\mu x_\mu$:

$$-\partial_\mu \partial_\mu h + V'(h) = -\frac{d^2h}{dr^2} - \frac{3}{r} \frac{dh}{dr} + V'(h) = 0, \quad (3)$$

and satisfies the boundary conditions

$$h'(0) = 0, \quad h(\infty) = v \rightarrow 0. \quad (4)$$

We can perform a tree-level computation of the tunnelling rate with a negative $\lambda < 0$ renormalized at some arbitrary scale μ . In this approximation, the tree-level bounce $h_0(r)$ can be found analytically and depends on an arbitrary scale R :

$$h_0(r) = \sqrt{\frac{2}{|\lambda|}} \frac{2R}{r^2 + R^2}, \quad S_0[h_0] = \frac{8\pi^2}{3|\lambda|}. \quad (5)$$

At first sight, computing the decay rate among two vacua in the approximation $V(h) = \lambda h^4/4$ where no vacuum exists may appear rather odd. However [4], the presence of a potential barrier around the false vacuum $h \sim 0$ is not necessary, since in quantum field theory the bounce is not a constant field configuration, and the energy in its gradient effectively acts as a potential barrier. Furthermore, the decay rate does not depend on the unknown physics that eventually stabilizes the true vacuum at $h \sim M_{\text{Pl}}$, if the bounce has size $R \gg 1/M_{\text{Pl}}$: once a tunneling bubble appears, the instability due to $V'(h(0)) \neq 0$ brings h down to the true minimum with unit probability. Formally, by performing the analytic continuation from Euclidean $r^2 = x^2 + t^2$ to Minkowskian $r^2 = x^2 - t^2$ space-time, the evolution inside the bubble is described by eq. (5) with $r^2 < 0$: h_0 reaches a singularity at $r^2 = -R^2$. Indeed our potential is unbounded from below.¹ In general, what happens inside the bubble does not affect the tunnelling rate nor the growth of the bubble: being an $O(4)$ -invariant configuration (i.e. the bounce depends only on r), its walls expand at the speed of light, so that what happens in the interior cannot causally affect the exterior.

The arbitrary parameter R appears in the expression of the SM bounce $h_0(r)$ because in our approximations the tree level SM potential is scale-invariant: at this level, there is an infinite set of bounces of varying size R , all with the same action $S_0[h_0]$.

Quantum corrections are the dominant effect that breaks scale invariance, and have been computed in [4]. At one-loop order, the tunnelling probability in the universe space-time volume V_U is then given by

$$p = \max_R [p(R)], \quad p(R) = \frac{V_U}{R^4} e^{-S}, \quad (6)$$

¹ In the usual case, with a potential with two minima, the bounce can be computed only numerically. The analytic continuation can be done by switching $r \rightarrow ir$ in eq. (3) at $r < 0$, and solving numerically. The qualitative behavior of the solution can be understood by noticing that this operation is equivalent to flipping the sign of V : h oscillates around the true minimum, reaching it at $r \rightarrow -\infty$, i.e. for asymptotically large times inside the expanding bubble.

where $S = S_0 + \Delta S_{\text{1-loop}}$ is the one-loop action: since the bounce is not a static configuration, corrections to both the potential part, as well as to the kinetic part of the action, must be taken into account [4]. To find the bounce configuration that extremizes S , it is enough to evaluate it along the family of tree-level bounces, h_0 in eq. (5), and minimize with respect to R . The result is roughly $S \approx 8\pi^2/3|\lambda(\bar{\mu} = 1/R)|$, i.e. one-loop corrections remove the tree-level ambiguity on the RGE scale $\bar{\mu}$ by fixing it to be the scale $1/R$ of the bounce. Since within the SM $\lambda(\bar{\mu})$ happens to run reaching a minimal value at $\bar{\mu} \sim 10^{16-17}$ GeV, tunneling is dominated by bounces with this size. A posteriori, this justifies having neglected the SM mass term, that gives a correction $\Delta S \sim (mR)^2 \ll 1$ to the bounce action, and suggests that gravity should be taken into account perturbatively.

3 Vacuum decay with gravity

We now extend the previous computation taking into account gravity [7]. In our case this is a potentially relevant effect, since gravity breaks scale-invariance and $1/R$ is just somewhat smaller than the Planck scale. One might worry that gravity can have dramatic effects, and that the decay rate starts to depend on the unknown depth V_{\min} of the true minimum of the SM potential.² This is not the case. Since the exterior geometry is the flat Minkowski space, the generic argument given in the non-gravitational case still holds: the bubble is an $O(4)$ -invariant solution and its walls expands at the speed of light, irrespectively of what happens inside.³

We recall from [7] the basic formalism needed for a quantitative analysis. We assume an Euclidean spherically symmetric geometry, $ds^2 = dr^2 + \rho(r)^2 d\Omega^2$, where $d\Omega$ is the volume element of the unit 3-sphere. The Einstein-Higgs action

$$S = \int d^4x \sqrt{g} \left[\frac{(\partial_\mu h)(\partial^\mu h)}{2} + V(h) - \frac{\mathcal{R}}{2\kappa} \right], \quad (8)$$

where $\kappa = 8\pi G$ and $G = 1/M_{\text{Pl}}^2$ with $M_{\text{Pl}} = 1.22 \cdot 10^{19}$ GeV, simplifies to

$$S = 2\pi^2 \int dr \left[\rho^3 \left(\frac{h'^2}{2} + V \right) + \frac{3}{\kappa} (\rho^2 \rho'' + \rho \rho'^2 - \rho) \right], \quad (9)$$

where $'$ denotes d/dr . The equations of motion are

$$h'' + 3\frac{\rho'}{\rho}h' = \frac{dV}{dh}, \quad \rho'^2 = 1 + \frac{\kappa}{3}\rho^2 \left(\frac{h'^2}{2} - V \right). \quad (10)$$

We can analytically include the effect of gravity, assuming $RM_{\text{Pl}} \gg 1$, by performing a leading-order expansion in the gravitational coupling κ :

$$h(r) = h_0(r) + \kappa h_1(r) + \mathcal{O}(\kappa^2), \quad \rho(r) = r + \kappa \rho_1(r) + \mathcal{O}(\kappa^2). \quad (11)$$

The action is

$$S = S_0 + 6\pi^2 \kappa \int dr \left[r^2 \rho_1 \left(\frac{h_0'^2}{2} + V(h_0) \right) + (r\rho_1'^2 + 2\rho_1\rho_1' + 2\rho_1 r\rho_1'') \right] + \mathcal{O}(\kappa^2). \quad (12)$$

² This is what one would naïvely guess from the results of [7] for the bounce action:

$$S_{\text{with gravity}} \approx S_{\text{without gravity}} / [1 + R^2 V_{\min} / M_{\text{Pl}}^2]^2, \quad (7)$$

i.e. the bubble does not exist if the true minimum has a large negative cosmological constant, e.g. $V_{\min} \sim -M_{\text{Pl}}^4$. However, eq. (7) holds within the thin-wall approximation [7], not applicable when V_{\min} is large and negative, and not applicable to the SM case we are interested in.

³ It is only an observer inside the bubble that experiences a large negative cosmological constant and consequently a contraction down to a big-crunch singularity [7], instead than an expanding bubble.

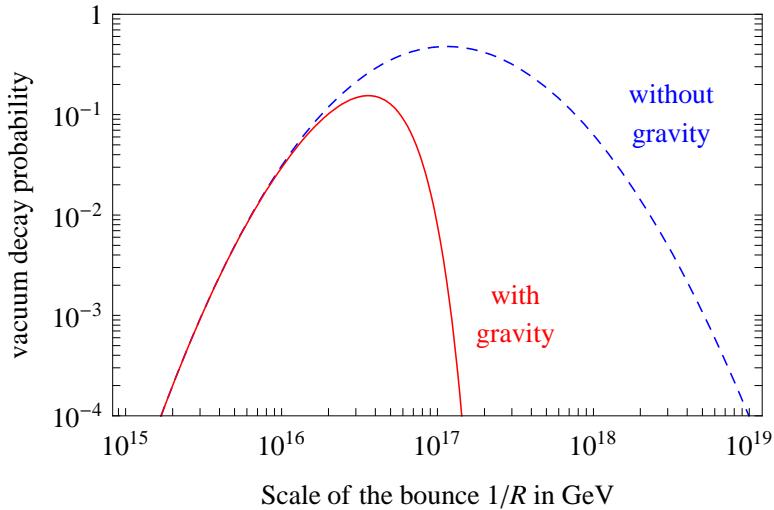


Figure 1: *Probability $p(R)$ that the SM vacuum decayed so far for $m_h = 115$ GeV, $m_t = 174.4$ GeV, $\alpha_3(M_Z) = 0.118$, due to bounces with size R , without including gravitational effects (dashed curve [4]) and including gravitational effects (continuous line). The correction is relevant only at $1/R \gtrsim 10^{17}$ GeV. Uncertainties due to higher-order corrections are not shown.*

We have taken into account that many terms in the expansion vanish either because the integrand is a total derivative (e.g. the negative power $1/\kappa$ in eq. (8) is just apparent) or thanks to the equations of motion. Indeed h_1 does not appear in eq. (12) because we are functionally expanding around the extremum h_0 of the non-gravitational action, so that the first functional derivatives vanish thanks to the equations of motion. So, we only need to compute ρ_1 : its equation of motion is

$$\rho'_1 = \frac{1}{6} r^2 \left(\frac{h_0'^2}{2} - V(h_0) \right). \quad (13)$$

Inserting it into eq. (12) completes the computation of gravitational corrections to leading-order in κ . We notice that the first term in eq. (12), which is linear in ρ_1 , contributes -2 times the last purely gravitational term in eq. (12), which is quadratic in ρ_1 . This happens because S must have an extremum at $c = 1$ under the variation $\rho_1(r) \rightarrow c\rho_1(r)$. The discussion is so far general, and by choosing toy potentials we verified that eq. (12) agrees with the full numerical result.

4 Vacuum decay with gravity in the Standard Model

Going to the SM case, using the analytic expression of eq. (5) for the bounce h_0 , we can perform all integrations analytically finding

$$S = \frac{8\pi^2}{3|\lambda|} + \Delta S_{1\text{-loop}} + \Delta S_{\text{gravity}}, \quad \Delta S_{\text{gravity}} = \frac{256\pi^3}{45(RM_{\text{Pl}}\lambda)^2} \quad (14)$$

where $\Delta S_{\text{gravity}}$ is the gravitational correction and $\Delta S_{1\text{-loop}}$ the one-loop correction, given in eq. (3.3) of [4]. Eq. (6) gives the tunneling probability $p(R)$.

Fig. 1 shows an example of the relevance of gravitational corrections. We checked that the leading-order approximation agrees with the result of a full numerical computation: eq. (14) correctly approximates the action of the true bounce, and the true bounce $h(r)$ is correctly approximated by $h_0(r)$ with the value of R that minimizes S .⁴

⁴ Here we comment about the comparison between the analytic result in eq. (14) and the full numerical computation.

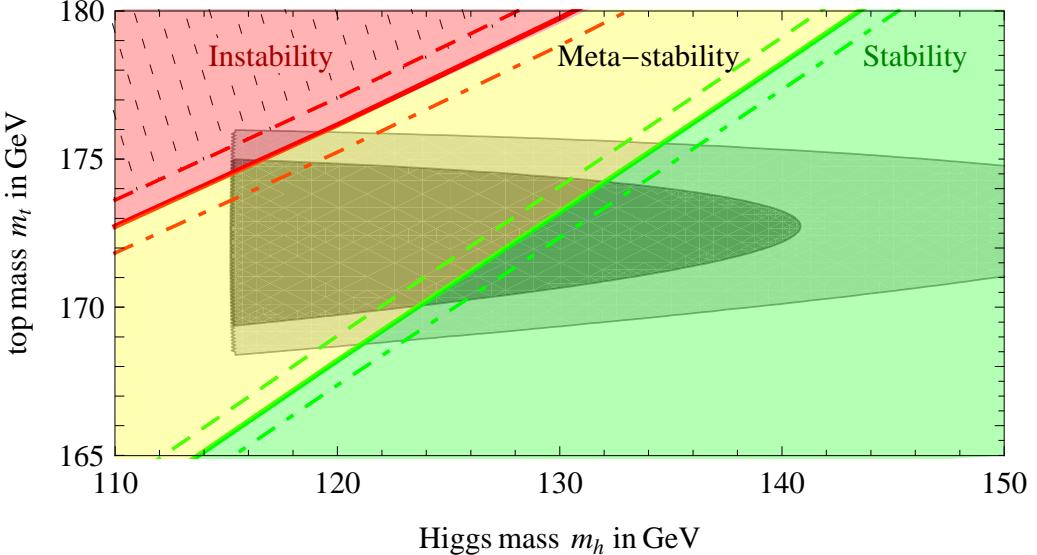


Figure 2: *Metastability region of the Standard Model vacuum in the (m_h, m_t) plane, for $\alpha_s(m_Z) = 0.118$ (solid curves). Dashed and dot-dashed curves are obtained for $\alpha_s(m_Z) = 0.118 \pm 0.002$. The shaded half-ellipses indicates the experimental range for m_t and m_h at 68% and 90% confidence level. Sub-leading effects could shift the bounds by ± 2 GeV in m_t .*

Fig. 2 shows the regions in the (m_h, m_t) plane where the SM vacuum is stable, meta-stable or too unstable. Gravitational corrections only induce a *minor shift* on the ‘instability’ border, less relevant than present experimental and theoretical (higher-order) uncertainties. The ellipses truncated at $m_h = 115$ GeV are the best-fit values for the top and Higgs masses, from our up-to-date global fit of precision data, that includes the latest direct measurement of the top mass, $m_t = (170.9 \pm 1.8)$ GeV [8]. Present data and computations indicate that we do not live in the unstable region (such that the SM can be valid up to the Planck scale), but increased accuracy is needed to determine if we live in the stable or in the small meta-stable region.

Adding to the SM action possible dimension-6 non-renormalizable operators suppressed by the Planck scale would give similar corrections to the bounce action. In particular, adding to the SM Lagrangian the operators

$$\Delta \mathcal{L}_6 = \frac{1}{M_{\text{Pl}}^2} \left(-\xi M_{\text{Pl}}^2 \mathcal{R} |H|^2 + c_1 \frac{|H|^6}{3!} + c_2 |H|^2 |D_\mu H|^2 \right), \quad (15)$$

where ξ and $c_{1,2}$ are unknown dimensionless coefficients, gives the following correction

$$\Delta S'_{\text{gravity}} = \frac{8\pi^2}{15(M_{\text{Pl}} R \lambda)^2} \left(128\pi\xi + \frac{c_1}{|\lambda|} + 4c_2 \right), \quad (16)$$

which can be comparable to the model-independent gravitational effect computed in eq. (14).

With a typical potential this is a straightforward procedure: the bounce is determined numerically as a compromise between classical solutions which under-shoot and over-shoot the true bounce at large r . With a potential close to h^4 , finding the bounce numerically is more involved: with this potential classical solutions necessarily go to zero at large r ; however, they generically oscillate to zero as $1/r$ giving a divergent action. The special feature of $h_0(r)$ is that it vanishes as $1/r^2$ giving a finite action. The true bounce should maintain this behavior. In practice, this is achieved imposing a vanishing difference between $h_0(r)$ and the numerical bounce. The advantage of our analytic approximation based on the set of candidate bounces $h_0(r)$ with different values of R is that ill-behaved never enter the computation.

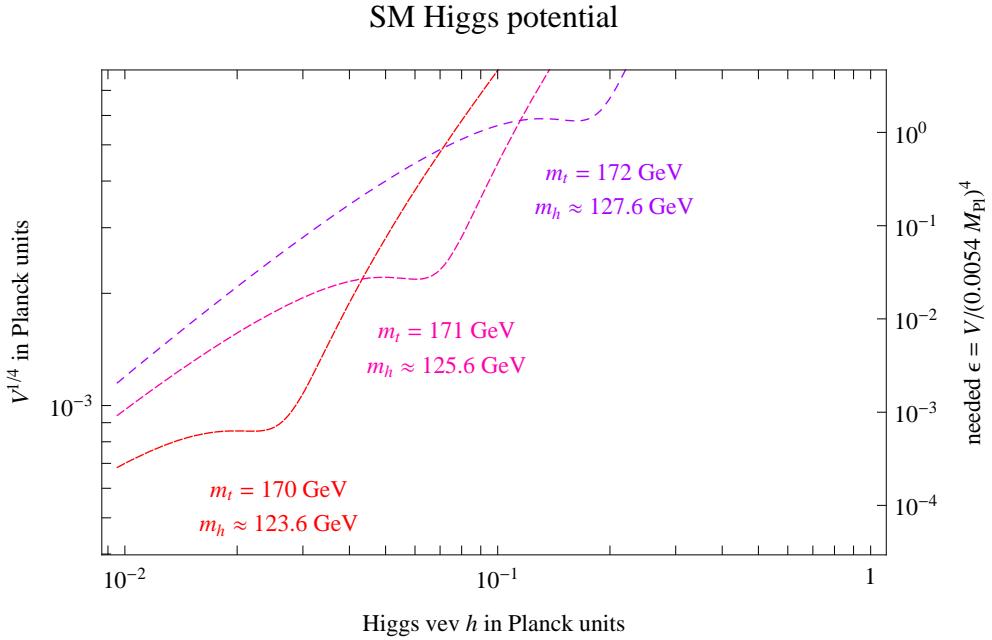


Figure 3: Examples of fine-tuned SM potentials that might allow inflation. The right handed axis shows the value of the slow-roll parameter ϵ that would give the observed amount of anisotropies.

The values of the coefficients ξ and $c_{1,2}$ change under field redefinitions and only their linear combination entering (16) is physical. Indeed, under $H \rightarrow H(1 + a|H|^2/M_{\text{Pl}}^2)$ we have⁵ $\delta c_1 = 24\lambda a$, $\delta c_2 = 6a$ and $\delta \xi = 0$; this transformation can be used to set $c_2 \rightarrow 0$. Under the Weyl transformation of the metric $g_{\mu\nu} \rightarrow g_{\mu\nu}(1 + a|H|^2/M_{\text{Pl}}^2)$ we have $\delta \xi = a/16\pi$, $\delta c_1 = 12a\lambda$, $\delta c_2 = a$; this transformation can be used to set $\xi \rightarrow 0$. Both these field redefinitions leave $\Delta S'_{\text{gravity}}$ invariant.

To estimate the magnitude of $\Delta S'_{\text{gravity}}$ we can thus restrict the attention to only one of the three operators in $\Delta \mathcal{L}_6$ (we choose the $|H|^6$ term), and estimate its coupling using naïve dimensional analysis. At one loop, graviton exchanges generate the $|H|^6$ operator with $c_1 \sim g_s^4/\pi$ as well as the $\lambda|H|^4$ operator with coefficient $\lambda \sim g_s^4/\pi^2$. Here g_s is an unknown coefficient which determines if quantum gravity is weakly or strongly coupled, with strong coupling corresponding to $g_s \sim \pi^2$. One might therefore argue that $c_1 \sim \lambda\pi$, which implies $\Delta S'_{\text{gravity}} \sim \Delta S_{\text{gravity}}$.

5 Inflation within the Standard Model?

For $m_t \approx 173$ GeV and $m_h \approx 130$ GeV (i.e. within the experimentally allowed region) both the quartic Higgs coupling λ and its β -function happen to vanish, at some RGE scale around M_{Pl} . Is this just a coincidence, or this boundary condition carries some message? Some speculations about this fact have been presented in [9]. Here we explore a different aspect, namely a possible connection with inflation.

The quasi-vanishing of both λ and $\beta(\lambda)$ allows to have a quasi-flat Higgs potential at $h \sim M_{\text{Pl}}$, suitable for inflation. Indeed, we can approximate the RGE running of λ as

$$\lambda(\mu \sim h_0) \simeq \lambda_{\min} + \frac{\gamma}{(4\pi)^4} \ln^2 \frac{\mu}{h_0} \quad (17)$$

around the special value h_0 where λ reaches its minimal value λ_{\min} . The constant γ is related to $\beta(\beta(\lambda))$ and has the numerical value $\gamma \approx 0.6$ within the SM. The first and second derivatives of the

⁵ We do not distinguish between $|H^\dagger D_\mu H|^2$ and $|H|^2 |D_\mu H|^2$ since these operators coincide on the configurations $H = (h/\sqrt{2}, 0)$ we are interested in.

SM potential $V \simeq \lambda(h)h^4/4$ vanish at $h = h_* \equiv h_0 e^{-1/4}$ if $\lambda_{\min} = \gamma/4096\pi^4$, such that the slow-roll parameters

$$\varepsilon \equiv \frac{M_{\text{Pl}}^2}{16\pi} \left(\frac{V'}{V} \right)^2, \quad \eta \equiv \frac{M_{\text{Pl}}^2}{8\pi} \frac{V''}{V}$$

vanish, allowing for inflation.

The lack of convincing natural models for inflation might indicate that it happens when scalar fields, fluctuating along some vast ‘landscape’ potential generically unsuitable for inflation, encounter a small portion of the potential which accidentally is flat enough. This is what might happen within the SM. This potential is illustrated in fig. 3, where we do not show the uncertainty due to higher-order corrections, which effectively amounts to a ± 2 GeV uncertainty in m_t . Can this SM potential be responsible for inflation *and* the generation of anisotropies $\delta\rho/\rho$? The answer is: not both. The basic problem is that the requirement of having enough *e*-folds of inflation,

$$N = 2\sqrt{\pi} \int \frac{dh/M_{\text{Pl}}}{\sqrt{\varepsilon}} \approx 60, \quad (18)$$

can be met with a small enough ε , but this conflicts with the requirement that quantum fluctuation of the Higgs inflaton should also generate the observed power spectrum of anisotropies, $\delta\rho/\rho \sim 10^{-5}$, i.e.

$$\frac{V}{\varepsilon} \approx (0.0054 M_{\text{Pl}})^4. \quad (19)$$

Indeed the height V of the SM potential in its flat region is predicted and cannot be arbitrarily adjusted to be as low as needed. This result can be understood by either doing explicit computations with the approximated potential $\lambda(h)h^4/4$, or by looking at the sample SM potentials plotted in fig. 3. For a top mass within the observed range, the plateau is at values of h and $V^{1/4}$ which are somewhat below the Planck scale, but $\delta\rho/\rho$ at $N \approx 60$ comes out larger than the observed value. Successful inflation and successful generation of anisotropies would be obtained if for some unknown reason the potential would remain flat from $h \sim h_*$ up to $h \sim M_{\text{Pl}}$.

6 Conclusions

In this paper we have refined and updated the metastability constraint on the Higgs mass, assuming the validity of the Standard Model up to the highest possible energy scale, $\Lambda \approx M_{\text{Pl}}$. In particular, we have taken into account gravitational corrections, which were neglected in previous analyses. These corrections turn out to be small and calculable in the phenomenologically interesting region of m_h close to its experimental lower bound. The updated constraints in the (m_h, m_t) plane are reported in fig. 2. Among all possible values, the Higgs mass seems to lie in the narrow region which allows the SM to be a consistent theory up to very high energy scales, with a perturbative coupling and a stable or sufficient long-lived vacuum. Fig. 4 illustrates the constraints on the Higgs mass as function of Λ , and shows that the (meta)stability constraints do not depend on Λ when it is around the Planck scale.

We have also shown that the SM potential can be fine-tuned in order to be made suitable for inflation. However, the resulting power spectrum of anisotropies is larger than the observed one.

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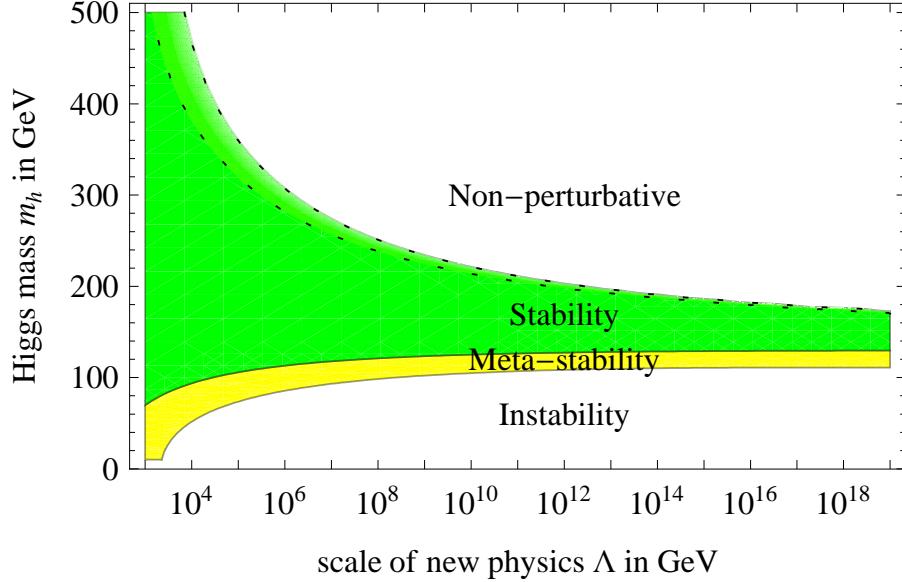


Figure 4: *Bounds on the Higgs mass derived by the conditions of absolute stability (lower bound), sufficient metastability (yellow region) and perturbativity (upper dotted lines, derived by the conditions $\lambda < 3, 6$), as function of the scale of validity of the SM. This plot assumes $m_t = 173$ GeV and $\alpha_3(M_Z) = 0.118$.*

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